Derivatives And Primitives Of The Hyperbolic Functions

DENIS BOBOCEA

Derivatives Of Functions

$$\frac{d}{dx}(\sinh(x)) = \cosh\,(x)$$

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$$\frac{d}{dx}(\tanh(x)) = \frac{1}{cosh^2(x)} = sech^2(x)$$

$$\frac{d}{dx}(\coth(x)) = \frac{-1}{sinh^2(x)} = -csch^2(x)$$

$$\frac{d}{dx}(\mathrm{sech}(x)) = -\tanh(\mathrm{x})\,\mathrm{sech}(\mathrm{x})$$

$$\frac{d}{dx}(\mathrm{csch}(x)) = -\coth\,(x)\mathrm{csch}\,(x)$$

Primitives Of Functions

$$\int \sinh(x)\,dx = \cosh(x) + C$$

$$\int \cosh(x)\,dx = \sinh(x) + C$$

$$\int \tanh(x)\,dx = \ln\,(\cosh(x)) + C$$

$$\int \coth(x)\,dx = \ln\,|\sinh\,(x)| + C$$

$$\int \mathrm{sech}(x)\,dx = 2\arctan(e^x) + C$$

$$= 2\arctan\left(\tanh\left(\frac{x}{2}\right)\right) + C$$

$$= \arctan(\sinh(x)) + C$$

$$\int \mathrm{csch}(x)\,dx = \ln\,(\tanh\left(\frac{x}{2}\right)) + C$$

$$= -2\,\mathrm{arctanh}(e^{-x}) + C$$

Derivatives Of Inverse Functions

$$\frac{d}{dx}(\mathrm{arcsinh}(x)) = \frac{1}{\sqrt{x^2+1}}$$

$$\frac{d}{dx}(\mathrm{arccosh}(x)) = \frac{1}{\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\mathrm{arctanh}(x)) = \frac{1}{1-x^2}$$

$$\frac{d}{dx}(\mathrm{arccoth}(x)) = \frac{1}{1-x^2}$$

$$\frac{d}{dx}(\mathrm{arcsec}\,(x)) = \frac{-1}{x\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\mathrm{arccsch}(x)) = \frac{-1}{x\sqrt{x^2+1}}$$

Primitives Of Inverse Functions

$$\int \mathrm{arcsinh}(x)\,dx = \mathrm{x}\,\mathrm{arcsinh}(x) - \sqrt{x^2+1} + C$$

$$\int \mathrm{arccosh}(x)\,dx = \mathrm{x}\,\mathrm{arccosh}(x) - \sqrt{x^2-1} + C$$

$$\int \mathrm{arctanh}(x)\,dx = \mathrm{x}\,\mathrm{arctanh}(x) + \frac{1}{2}\ln\,(1-x^2) + C$$

$$\int \mathrm{arccoth}(x)\,dx = \mathrm{x}\,\mathrm{arccoth}(x) + \frac{1}{2}\ln\,(1-x^2) + C$$

$$\int \mathrm{arcsech}(x)\,dx = \mathrm{x}\,\mathrm{arcsech}(x) - 2\arctan\,(\sqrt{\frac{1-x}{1+x}}) + C$$

$$\int \mathrm{arccsch}(x)\,dx = \mathrm{x}\,\mathrm{arccsch}(x) + \arctan\,(\frac{x}{\sqrt{x^2+1}}) + C$$